TOPOLOGY II BACKPAPER EXAM

Attempt any SEVEN questions, all questions carry equal marks.

(1) Show that $S^1 \times S^1$ and $S^1 \vee S^1 \vee S^2$ have isomorphic homology groups in all dimensions but their universal covering spaces do not.

(2) Compute the homology groups of all the nonorientable compact surfaces N_g (here, N_g is the connected sum of g copies of the real projective plane).

(3) Describe a CW complex structure on the complex projective spaces $\mathbb{C}\mathrm{P}^n$ (for all $n \geq 1$), and compute their homology groups using the associated cellular complex.

(4) Given any positive integer n, construct a path connected topological space X such that $H_n(X) \cong \mathbb{Z}/2\mathbb{Z}$ and all other reduced homology groups of X are trivial.

(5) Compute the homology groups $H_i(\mathbb{R}\mathrm{P}^2; \mathbb{F}_p)$ of $\mathbb{R}\mathrm{P}^2$ with coefficients in the finite field $\mathbb{F}_p$ with p elements (where p is any prime integer).

(6) Compute the relative homology groups $H_i(S^3, S^2)$ where S^2 is the "equator" of S^3 ($S^2 = \{(x, y, z, w) \in S^3 | w = 0\}$).

(7) Draw pictures and describe two path connected covering spaces of the figure eight, one of which is regular and the other which is not. Justify your answers.

(8) What is the fundamental group of $\mathbb{R}\mathrm{P}^2 \vee \mathbb{R}\mathrm{P}^2$? What is its universal covering space?

(9) What is the fundamental group of a Mobius strip? Draw all connected covering spaces of it.

(10) What is the fundamental group of a Klein bottle? What is its universal covering space? Draw a non trivial covering map of the Klein bottle which is not the universal covering map.